Longitudinal Equations of Motion[Sa1]

The longitudinal equations of motion for an electron in a storage ring are

$$\frac{ds}{dt} = -\alpha\varepsilon c, \qquad \& \qquad \frac{d\varepsilon}{dt} = \frac{eV_{RF}(s) - U(\varepsilon)}{E_0 T_0},$$

where $\varepsilon \equiv \Delta p/p$ and s are the momentum deviation and distance of the electron from the synchronous particle respectively. Note that s is positive when an electron arrives at each azimuth ahead of the synchronous particle. If the RF voltage, V_{RF}, is assumed to be sinusoidal the following quantities are of interest.

1. Synchronous Phase, ϕ_s:

$$\phi_s = \sin^{-1}\left[\frac{U_0}{eV_{RF}}\right] = \sin^{-1}\left[\frac{1}{q}\right]$$

2. RF Acceptance, ε_{RF}:

$$\varepsilon_{RF} = \pm\left[\frac{2U_0}{\pi\alpha hE}\left\{\sqrt{q^2 - 1} - \cos^{-1}(1/q)\right\}\right]^{1/2}$$

3. Synchrotron Tune, ν_s:

$$\nu_s = \frac{\Omega_s}{\omega_0} = \left[\frac{\alpha h\cos\phi_s}{2\pi}\frac{eV_{RF}}{E}\right]^{1/2}$$

4. Bunch Length, σ_L:

$$\sigma_L = \frac{\alpha c}{\Omega_s}\sigma_E = \left[\frac{2\pi\alpha hc^2}{\omega_{RF}^2\cos\phi_s}\frac{E}{eV_{RF}}\right]^{1/2}\sigma_E$$